METEORIC INFALL AND LUNAR SURFACE ROUGHNESS

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ABSTRACT

Assuming that the present meteoritic infall as measured on the earth has persisted for the age of the lunar maria, it is possible to calculate the number density of small craters on the lunar surface. Use of estimated errors generates three number diameter relations of confidence levels crudely estimated as 10, 50, and 90%. The nominal (50%) density of craters exceeding 1 meter in diameter is .06 per square meter; the other relations are about 20 times higher and lower.

Under the hypothesis that erosion by micrometeoroids results in a protective debris layer, calculations indicate that this layer is expected (50%) to be less than 0.1 meter thick, with a 6x span to the 10% and 90% levels. Any primordial lunar relief would be eroded by a smaller amount. The debris layer will be much more substantial near larger craters.

Finally, arguments assuming a fairly general comminution law suggest that the number density of large ejected fragments on the surface should be similar to that of craters of equal volume.

Since other processes than meteoritic infall are probably important in determining lunar topography, one must be cautious in asserting that these results in fact describe the moon.

I. INTRODUCTION: SURFACE MODELS

The question of the nature of the lunar surface arises in almost every study related to the Apollo lunar landing mission. These studies include not only the landing of the space ship itself, but also, the design of surface vehicles, the plans for manned exploration, and the design of photographic reconnaissance equipment. Regretfully, however, years of intensive study have if anything obscured our concept of the lunar surface. Until some direct information is obtained, these essential studies have no alternative but a group of plausible surface models, of varying inhospitality.

A proposed landing vehicle, for instance, can be judged on the number of such models with which it can deal; only when more is known can it be designed for excellence in coping with a particular model. An effort to produce a group of statistical lunar surface models is in process. The present paper, dealing with the effects of meteoric bombardment, is a portion of that effort.

The gross appearance of the moon is dominated by craters generally believed to be of meteoritic origin. The crater density is unknown for diameters under half a kilometer or so, but there is considerable information on the terrestrial infall of meteorites which would produce craters on the scale of 0.1 to 100 meters in diameter. This information is reviewed below, and a best estimate of the crater population is made. We further attempt to estimate the numbers of crater fragments of various sizes, and the extent to which these features have been modified by meteoric erosion. Some of the data is very poor. Recognizing this, we attempt to estimate errors. At each step in the argument where a numerical value must be postulated, a logarithmic probable error is assigned. It is intended that the true values have a chance exceeding 80% of falling within the error interval. The errors are assumed independent, and are propagated through the computations accordingly, the fractional error in a product being the Pythagorean sum of the errors in each factor. Thus, if e_x is the error in x, and similarly for y and z, the error in the product xyz is:

$$e_{xyz} = \sqrt{e_x^2 + e_y^2 + e_z^2} \qquad (1.1)$$

Thus, the finished calculation contains an optimistic, a nominal, and a pessimistic case. These should be considered upper bounds on the results, with confidences of 10%, 50%, and 90%, respectively. These levels are subject to revision, and are stated only to set orders of magnitude. The set can readily be used in a sequence of surface models. The chances that the proposed models statistically describe the "real moon" are problematic; certainly they are good only if meteoric infall is the major influence on small scale topography. Other processes, including vulcanism, may produce a rougher terrain, or a smoother one. A quantitative model, however, must be held in considerable respect, particularly since, as will be seen, the inferred surface is more irregular than has been proposed in the past.

II. CRATER COUNTS AND METEORIC INFALL

A. CRATERS

To the naked eye observer, the moon is divided into bright (continental) and dark (mare) areas. With even a small telescope, the craters become apparent. Substantial numbers appear on the maria, and the continents are literally covered with these mammoth, circular features.

The general nature of the craters has been discussed in a number of books. (1) They are generally round. Large craters are shallow; their floors seem to follow the mean surface of the moon. Smaller craters may be quite deep. As examples, Copernicus has a ratio of rim diameter to total depth of 90 km/3.5 km = 26:1. Lalande A (10°W, 7°S) is 12 km/2.5 km = 5:1. These figures are taken from the Air Force Lunar Charts. (2) The diameter-depth ratio for fresh, small craters may be 4:1. This figure is suggested by H. J. Moore (3) of the U. S. Geological Survey.

The Survey's Astrogeologic section (4) and others have shown that craters may be classified by relative age. At least five age classes can be distinguished; the walls of the old craters are degraded, and pitted by younger craters. The youngest craters of all are characterized by sharp contours, rubbly surrounding material, and by bright, more or less radial "rays" which may (particularly for the crater Tycho) span the entire visible surface of the moon.

To order the craters by age is not to assign calendar ages. In the point of view taken in this paper, the craters are a result of the moon's collisions with other, smaller celestial bodies. The continental regions may represent the concluding stages of the consolidation of the solar system. Subsequent to this, the great maria were formed. Several of these are quite circular, including Mare Imbrium, 500 km across. These also may have resulted from impacts, and if so, exceedingly spectacular ones. The maria now appear smooth and reasonably flat. As for the surface material, its nature is unknown.

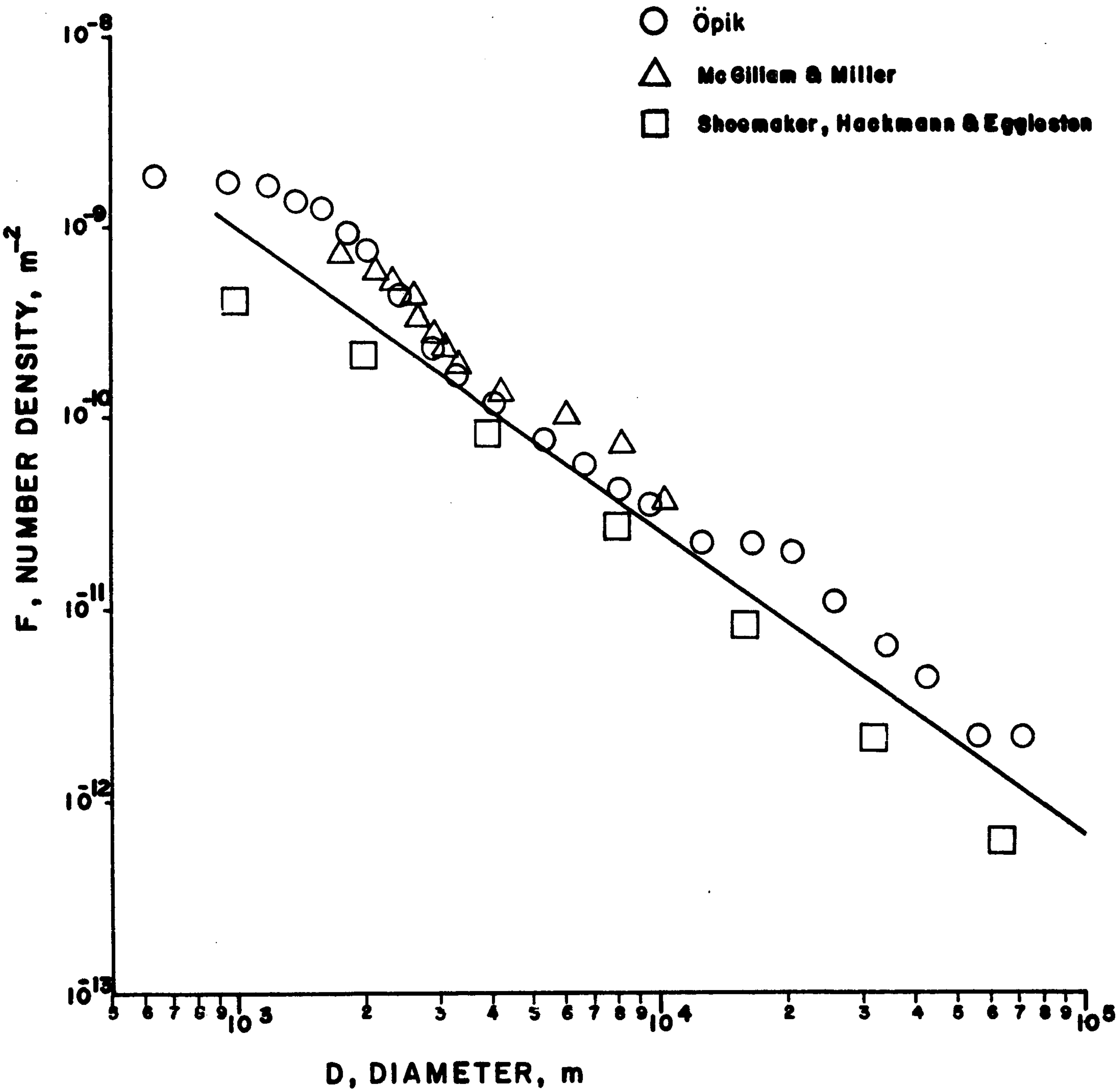
In any case, the maria are "new" relative to the continents. The age we use below is $10^{9\pm0.7}$ years - not younger than 200 million years, and no older than the proposed 4.5 billion year age of the earth and the solar system. On the maria, the crater density is fifteen-thirty times smaller than on the continents.[8] We hypothesize that at least the marial craters have accumulated uniformly over the years since mare formation.

Today, the collision or impact hypothesis for the origin of the craters is widely held. Baldwin[5] has been a forceful exponent for many years. Divergent hypotheses generally posit a volcanic origin for some or all of the features. This idea is particularly compelling for non-random arrays of small craters, or for craters associated with linear features or domes.

The majority of the craters seem, however, to be randomly placed. An impact origin for these is further supported by the reasonable matching of the density of small craters with the known, current, infall rate of substantial meteoroids, and with the number of comparable features on the earth.[6]

Let us now briefly review some of the published "crater counts" of number versus diameter. We restrict ourselves to mare regions. The investigators start with a high resolution photograph (6-Shoemaker, Hackmann & Eggleston, 7-Opik) or the Air Force Charts (8-McGillam & Miller) and tabulate the observed features. The Geological Survey paper [6] has a good discussion of features which should be excluded. These include craters which are probably volcanic, and "gouge-like depressions," "probably secondary impact craters formed by fragments ejected from the large craters."

On Figure 1, the data from the three investigations is plotted. These have been expressed as the cumulative number of craters exceeding diameter D (in meters) per square meter. Otherwise, every quoted number is included. It will be seen that Opik [7] and McGillam [8] counts are



CRATER COUNTS

FIG. 1

consistent (within a factor of two total range) and that the Geological Survey count,[6] with its deliberate omissions, is low. The deviation may be statistical at the large diameter end, but in the mean, amounts to about three times.

We have chosen the straight line on Figure 1 as a suitable representation of the data. Within a factor of two, it includes most of the points. If F is the number density (m^{-2}) and D the crater diameter (m), the equation of this line is:

$$\log F = -4.28 - 1.58 \log D \pm 0.3 \qquad (2.1)$$

$$3 < \log D < 5$$

In this paper, "log" will indicate the common logarithm, and "ln" the natural logarithm.

Exponents from 1.5 to 1.7 would fit, and the general applicability can be judged by the points on Figure 1.

Are these crater densities consistent with the meteoric infall hypothesis? It will be seen that they are, in that they are consistent with the terrestrial infall rates observed today. The hypothesis can then be advanced that these same infall rates should be used to compute the number densities of smaller lunar craters, to extend Figure 1 down to the 1 meter size range.

B. METEORIC INFALL: THE METEOROIDS

In an earlier paper, the author has described the Meteoroid Environment of Project Apollo,[9] and some of the background material there may be of use. In fact, however, the present investigation is complementary. The small particles just capable of penetrating spacecraft skins would make negligible craters on the moon. The particles which leave substantial craters (10 centimeters or more in diameter) are too rare to affect Apollo spacecraft design or mission strategy.

The meteoroids which concern us are largely the museum meteorites, dense bodies of composition and structure suggesting that they once formed part of a substantial planetoid. Perhaps 10% of the commonly found meteorites are nickel-iron. The rest, the stones, have a wide variety of compositions. G. S. Hawkins[10] states that the heaviest meteoroids (above 10^6 kg) are predominantly irons.

These bodies move in the solar system predominantly in direct orbits (i.e. in the same sense as the earth) of varying eccentricity and their impact velocities vary accordingly. Hawkins [10] gives 17 km/sec as a mean velocity for impact with the earth. Compensating for the respective escape velocities, this should amount to about 13 km/sec for impacts on the moon. This mean velocity is rather smaller than that observed for the more common meteoroids, which have a higher percentage of retrograde orbits.

C. HYPERVELOCITY IMPACTS

When a meteoroid collides with a solid surface, the result is analogous to an explosion. The impact velocity is substantially above the sound velocity in the target. Essentially the entire kinetic energy of the meteoroid is imparted to the target in a localized, highly shocked region; crater formation occurs subsequently, with the dissipation of this energy.

Studies of cratering are reported, for instance, by Baldwin [5] and Shoemaker et at. [6] More general studies of hypervelocity impact are contained in the proceedings of the Hypervelocity Impact Symposia. [11]

There is considerable disagreement among the experts (see (9), (24), and other review papers). Some groups associate crater volume with the kinetic energy of the projectile; others [6] choose, for larger craters, perhaps the .88 power of kinetic energy. Still others incline to a dependence of crater volume on projectile momentum. [12] Experimental impact experiments do not cover velocities much in excess of 10 km/sec, so that, particularly for cometary meteoroids, substantial extrapolation error is introduced. Since the mean impact velocity of heavy meteoroids on the moon is only 13 km/sec, the question of velocity dependence is less important for the present study.

On the other hand, the question of material strength is important. Unfortunately we have no knowledge of the strength of lunar surface material. Briefly, the early stages of impact are characterized by pressures of millions of atmospheres. The strength and shape of projectile and target are not important. During the "explosive" or terminal stages of cratering, however, the strength of the target almost certainly determines the ultimate crater size and shape. Unfortunately, we cannot estimate the strength of the lunar surface. This uncertainty controls the usefulness of our cratering criterion, and makes the precise choice immaterial.

In other studies(9), we have used the Charters and Locke(13) penetration relation, which is expressed as,

$$\frac{p}{d} = 2.28 \left(\frac{\rho_p v}{\rho_t c} \right)^{2/3} \tag{2.2}$$

Here p is the penetration in a semi-infinite solid of density ρ_t and sound velocity c, resulting from the impact of a particle of diameter d, density ρ_p, and velocity v. Scaling material properties with sound velocity is frankly successful only for a few metals; it is adequate for order-of-magnitude studies. Other criteria (see reference 9) often introduce the "crushing strength", S of the target. One may substitute $c^2 = S/\rho_t$ with alterations of the constant. Cratering relations used by Shoemaker(6) and Baldwin(5) predict craters in soils larger by perhaps 10x in volume. Relation (2.2) is as adequate as any to start from.

We would prefer a relation between crater diameter, D, in meters, and particle mass, m, in kilograms.

We assume that the Charters and Locke relationship adequately predicts the volume ejected from a crater, and that this volume is $\frac{2}{3} \pi p^3$. Then,

$$(2p)^3 = 181 \left(\frac{\rho_p}{\rho_t} \right) \frac{m v^2}{\rho_t c^2}, \tag{2.3}$$

clearly dependent on particle kinetic energy. A private communication from H. J. Moore of the U. S. Geological Survey suggests that real impact craters are probably less than hemispherical, with diameter to depth ratios of about 4:1. Thus, the diameter of the crater will be rather larger than 2 p. Define R, the ratio of the volume of a hemisphere to that of the shallower crater of equal diameter. Equation (2.3) then becomes

$$D^3 = 181 \left(\frac{\rho_p}{\rho_t} \right) R \frac{m v^2}{\rho_t c^2}. \tag{2.4}$$

For the lunar surface, we choose $\rho_t = 3.5 \times 10^3$ kg/m^3, and C = 5 x 10^3 m/second. The first is the approximate bulk density of the moon; the latter a typical value for strong solids. These values probably overestimate the strength of the moon; they are certainly only applicable to impacts on the lunar "bedrock". Impacts on a porous surface layer will be treated later. With these values, it appears that a one kilogram object of density equal to that of the moon, striking at 20 kilometers per second, would make a hemispherical crater of diameter one meter. This is shown in,

$$D^3 = \left(\frac{\rho_p}{\rho_t}\right) R\ m \left(\frac{v}{2 \times 10^4}\right)^2 \tag{2.5}$$

For a spherical cap of 4:1 diameter/depth ratio, R ~ 2.5. Equation (2.5) will be used below to estimate the crater diameters resulting from given impacts. The expected error is substantial; at 15 km/sec, Bjork[12] would predict 3x smaller values for metallic impacts; as stated, Shoemaker[6] might predict 10x larger values.

We assign a logarithmic probable error of ± 0.8, or about six times, in D^3.

D. METEORIC INFALL: FLUXES

The meteoric data falls in three groups: first the cometary meteors, which are important in the smaller size ranges; second, the bright visual meteors, or fireballs; and third, the meteorites, the heaviest of these objects, which survive passage through the earth's atmosphere and are found on the ground.

Cometary Flux. These particles form the majority of the "visual meteors." They are frequent enough to be effectively counted by patrol cameras. The classical report in this field is that of Hawkins and Upton.[14] Twin cameras with interrupting shutters are placed on a 40 km baseline. Altitude, velocity, and meteor brightness can be measured. As discussed throughout the literature [see (9), but for original work E. Opik[15] and B. I. Levin[16]] the instantaneous brightness can be tied quantitatively with the instantaneous ablative mass loss of the meteoroid. The integrated light curve is then a measure of mass. Relative masses are considered quite reliable; the absolute values are in considerable doubt. Hawkins and Upton stated the results of their survey as:

$$\log N = -1.34 \log m - 2.64 \tag{2.6}$$

N is the cumulative influx of meteors exceeding a mass m (grams) per square km, per hour. The zero magnitude meteor was taken as 30 grams in weight, entering the earth's atmosphere at 30 km/sec. Currently, F. L. Whipple[17] regards 1 gram as a preferable value; Hawkins inclines to nearer 4 grams; with an uncertainty of 5x. The extreme values - 30gm, or .05gm (Levin[16])- result from specific traceable assumptions. A range of about 0.2 - 5 grams is unresolved.

Our model for the cometary influx on the moon is as follows: We assign a 1gm zero magnitude mass in equation (2.6) and express it in m.k.s units, obtaining*

$$\log N = -1.34 \log m - 18.20 \quad m^{-2}sec^{-1}, kg \quad (2.7)$$

The uncertainty is taken as Hawkins 5x in mass. Logarithmically, this will appear in (2.7) to be $\pm 1.34 \cdot 0.7 = \pm 0.94$. The mean velocity impacting the moon, with its lower gravitational potential, is 28 km/sec rather than 30, but it seems unnecessary to make this change.

The density of the cometary meteoroids is small; Whipple(17) chooses 0.44 gm/cc. For simplicity, we prefer 0.5 gm/cc. We make no allowance for the fact that the moon, as a smaller body, will capture somewhat fewer meteoroids per unit area than the earth.

Fireballs. Fireballs are the brighter visible meteors. Observational data was summarized by Hawkins in reference (18). This article has been superseded by more recent ones (10,19) in which he considers, as well, catalogs of meteorites. He seems to extrapolate with considerable enthusiasm, and his mass range (1 to 10^{14} kg) is probably broader than is justified. In any case, he quotes:

The flux of stony meteorites (density ~ 3.5):

$$\log N = -3.73 - \log m \quad (2.8)$$

The flux is per km^2 year, the mass in kilograms. This is derived from the catalogs of recovered meteorites assuming that 90 % of a stone is ablated in the earth's atmosphere. This 90% figure seems excessive for very large meteoroids. Levin ((16), p. 145) states that for slow, large meteoroids, as much as 90% of the mass may survive! We feel that Hawkins' masses are overestimated for large particles, and reduce the mass flux by 3x. We will then allow a mass uncertainty of 3x, bracketing ablations of 0 to 90%. The equation becomes (since log 1 km^2 year = log m^{2}sec + 13.50):

$$\log N = -17.73 - \log m \quad (2.9)$$

in m^{2}sec, kg.

The uncertainty here is, then, 3x in mass (dependent on ablation) and probably 4x in flux as will be discussed

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*This is the mass flux of the O.M.S.F. Program Directive, Natural Environment and Physical Standards for Apollo. M-DE 8020.008A August 15, 1963 (conf).

below. In accordance with (1.1), the expected logarithmic error is $\sqrt{.36 + .23} = 0.77$.

For iron meteoroids, Hawkins quotes

$$\log N = - 5.61 - 0.7 \log m \quad (km^2 yr,\ kg) \qquad (2.10)$$

The iron meteoroids are less populous than the stones in the kg range, but because of the smaller coefficient, (0.7), dominate the largest size ranges. He assumes 80% of an iron is ablated (the mass in space exceeds the observed mass by 5x). As before, we reduce this to (logarithmically) 0.35 ± 0.35.

$$\log N = - 19.36 - 0.7 \log m \pm 0.65 \quad (m^2 sec,\ kg) \qquad (2.11)$$

The density of iron is taken 8 gm/cc; the modal impact velocity for irons and stones on the moon, as 13 km/sec. The error is taken as 4x in flux and $\sqrt{5}$ in mass. The uncertainty in equation 2.11 is $\pm$ 0.65.

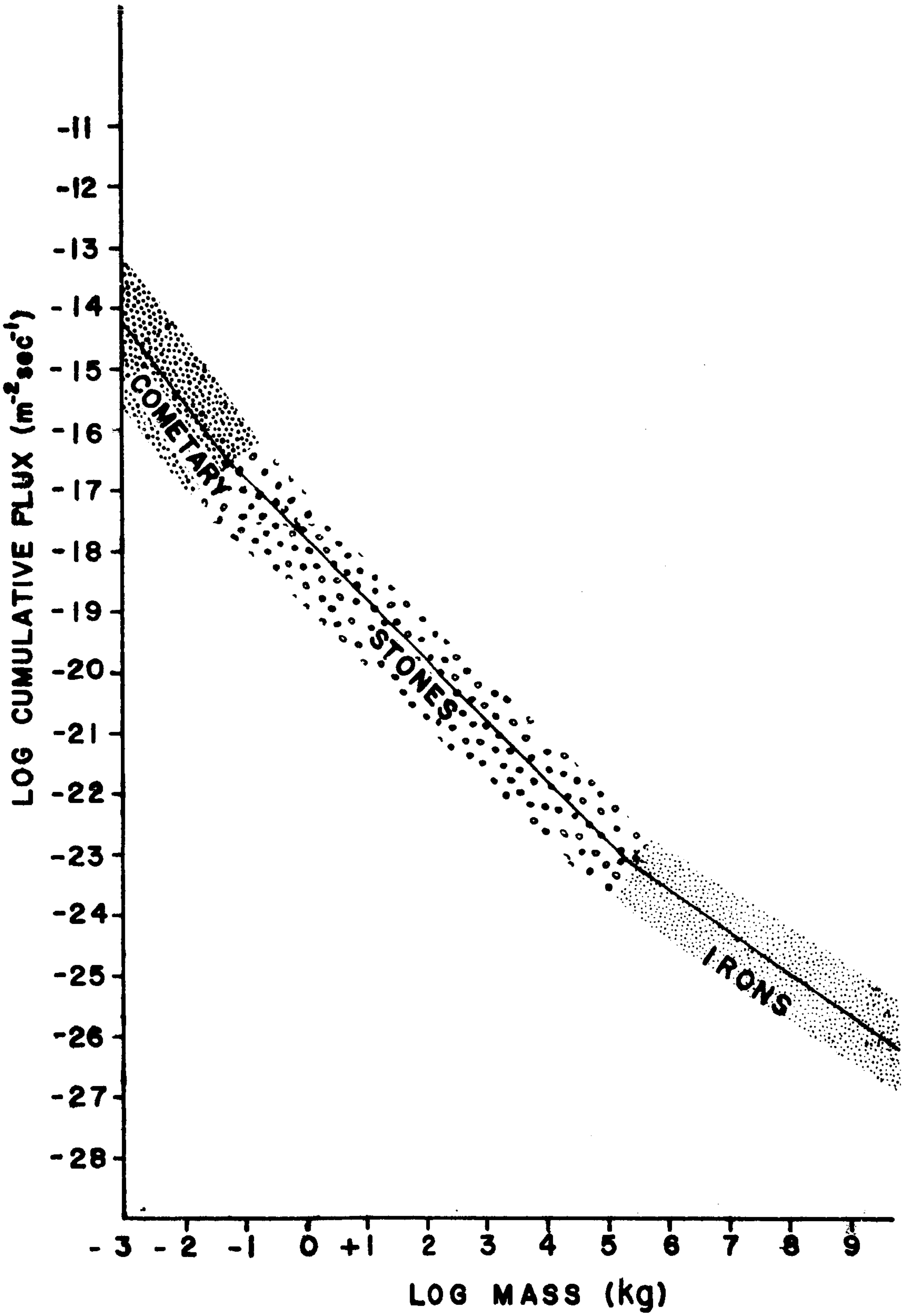
Meteorites. The Uncertainty in Flux. The articles by Harrison Brown(20) and an associate, Hugh Millard[21], are invaluable as studies in the certainties or uncertainties of meteorite data. To reflect accurate flux data, only witnessed "falls" are counted. For any degree of accuracy, only regions of very dense population are usable. The "collection efficiency" for falls varies with time of day and with the season. The Millard study, which includes a quantitative model of the collection process, concludes that the observed number of falls is 15 times low.

These papers are very competent and well documented. Brown's count (with no allowance for ablation) is, approximately,

$$\log N = - 18.8 - 0.76 \log m \quad (m^2 sec,\ kg) \qquad (2.12)$$

Brown does not recognize any difference of slope between stones and irons. Even with allowance for ablation, this is 4x lower than Hawkins estimate at the 1 kg level, and 15x lower than Millard would prefer. With this information we assign 4x (i.e., about $\sqrt{15}$) as the uncertainty in the flux of meteorites.

The flux mass relations described above are plotted on Figure 2. They show a gradual decrease in slope as mass



CUMULATIVE METEOROID FLUX
AGAINST MASS
FIG. 2

increases. A steep slope corresponds to a population which has been repeatedly fragmented - or perhaps which is very fragile. A shallow slope corresponds to "under-grinding" or, for very large masses, to accretional processes. According to Hawkins, the shallow slope for "irons" is due to their strength, and thus their resistance to "grinding". We shall now apply the chosen cratering law to these flux relations, and estimate crater densities.

E. INFALL AND CRATER DIAMETERS

The meteoric infall described above certainly makes craters today. The crucial assumptions we now make are: (a) that the influx has persisted unchanged since mare formation on the moon, and (b) that this time is $10^{9 \pm 0.7}$ yr ($10^{16.5 \pm 0.7}$ seconds).

On the first point, it is necessary to assume a steady state, with meteoroids being supplied from a reservoir and depleted. There is reasonable support for this. Whipple [22] quotes lifetimes for cometary meteoroids as short as 10^4 years. Hawkins [19] calculates that the total asteroidal (stone and iron meteorite) infall on the earth is a negligible drain on the asteroid belt. If the flux is not reasonably constant, it has most probably been decreasing; the resulting estimates of crater density will be small.

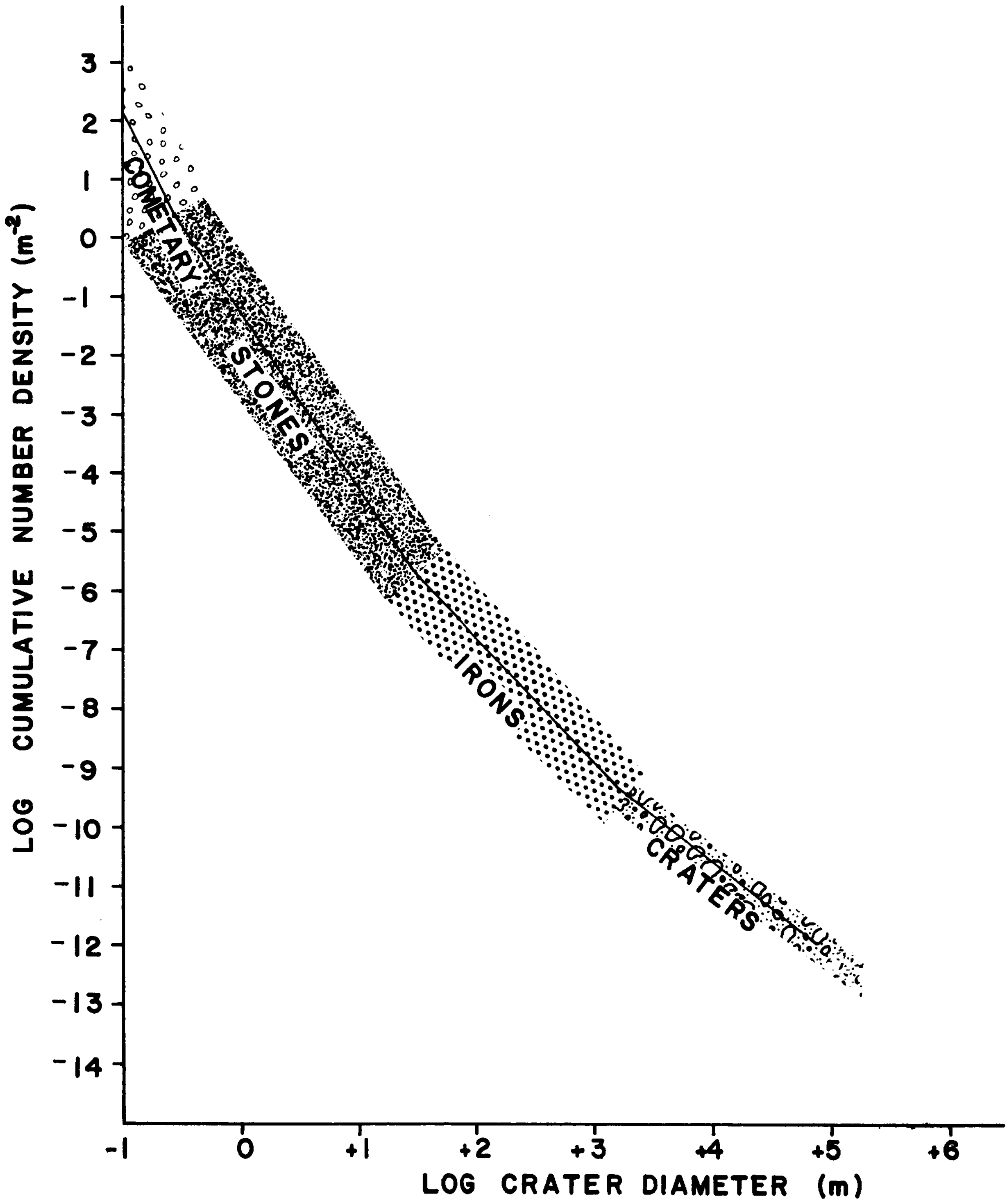
The estimate of the age of the maria (200 million years to five billion years) brackets various proposed values. The lower figure arises from assuming a constant meteoroid infall rate and, judging by relative populations, estimating the maria as younger than the continents in the same proportion. The continents are assigned an age of 4.5 billion years, and the 200 million year figure results.

The cratering relation (2.5) is now applied to the various fluxes. Crater cumulative number density, $F(m^{-2})$ is obtained from the fluxes using:

$$\log F = \log N(D) + 16.5 \pm 0.7 \qquad (2.13)$$

For neatness, this work is summarized in Table I. The results are shown on Figure 3, together with the crater counts. The matching of the nominal values is gratifying, but it is only significant within the proposed error ranges.

The agreement <u>is</u> entirely adequate to encourage the use of the infall rates to estimate small crater populations. Over the range of around 0.1 to 10 meter in crater diameter,



CUMULATIVE LUNAR CRATER DENSITIES,
OBSERVED AND INFERRED

FIG. 3

TABLE I

CRATER NUMBER DENSITY RELATIONS

Assumption:

$$3 \log D = \log \left(\frac{\rho_p}{3.5}\right) + \log m + 2 \log \frac{v}{2 \times 10^4} + \log R \pm 0.8 \qquad \text{from (2.5)}$$

Crater Number Density: $\log F = \log N + 16.5 \pm 0.7$ (2.13)

$R = 2.5$ (4:1 craters)

Cometary Infall: ($v = 3 \times 10^4$ m/sec, $\rho_p = 0.5$ gm/cc)

$$\log N = - 1.34 \log m - 18.20 \pm 0.94 \qquad (2.7)$$

$$4.02 \log D = 1.34 \log m - .13 \pm 1.07 \text{ from (2.5)} \qquad (2.14)$$

$$\log F = - 4.02 \log D - 1.83 \pm 1.59 \qquad (2.15)$$

Stone Infall: ($v = 1.3 \times 10^4$ m/sec, $\rho_p = 3.5$ gm/cc)

$$\log N = - \log m - 17.73 \pm 0.77 \qquad (2.9)$$

$$3 \log D = 0 + \log m + .02 \pm 0.8 \qquad (2.16)$$

$$\log F = - 3 \log D - 1.21 \pm 1.31 \qquad (2.17)$$

Iron Infall: ($v = 1.3 \times 10^4$ m/sec, $\rho_p = 8$ gm/cc)

$$\log N = - 19.36 - 0.7 \log m \pm 0.65 \qquad (2.11)$$

$$2.1 \log D = 0.7 \log m + .268 \pm 0.56 \qquad (2.18)$$

$$\log F = - 2.1 \log D - 2.59 \pm 1.18 \qquad (2.19)$$

TABLE I (continued)

For stone and iron,

Empirical Range: $1 - 10^4$ kg, or $1 < \log D < 1.33$

Est. Range (Hawkins $1 - 10^{14}$ kg) or $\log D < 4.7$

Crater Density:

$$\log F - 4.28 - 1.58 \log D \pm 0.3 \qquad (2.1)$$

$$3 < \log D < 5$$

it is the "Hawkins stone" population which dominates, with a slope of -3 and a density of one crater exceeding 1 meter diameter every 16 square meters, with a 20x uncertainty. We have indeed generated three dissimilar models. The optimistic case, (one crater every 240 sq m) is none too smooth. The pessimistic case would appear nearly as rough to the astronaut as the continental regions do in a small telescope.

This estimate of small crater density is high, compared with several others we have seen. For example, using the observed lunar crater population (2.1), one can extrapolate to the 1 m level, obtaining one crater one meter or greater every 30,000 m^2. This ignores the known meteoric infall; if there is firm information available, it is in the slopes of the distribution on Figure 3. The infall steepens as one goes to small masses, and the crater density - if it was formed by infall - must do likewise for small craters. The magnitudes are in doubt, but the upward trend is certain.

Other questions which arise immediately are: Does not the erosive action of the meteoroids greatly reduce this relief? What is the distribution of the debris ejected from the craters? What is the significance of these estimates for spacecraft operations? These questions will be dealt with - insofar as practical - in the next three chapters.

III. METEORITIC EROSION - DUST LAYERS

Webster's Third International Dictionary describes "erosion" (meaning #2) as "the general process whereby the materials of the earth's surface are worn away and removed by natural agencies including weathering, solution, corrasion and transportation." In this sense, every crater-forming impact is erosive, including Copernicus and Kepler. More specifically, however, we want to deal with a general, omnipresent erosion to which every area of the moon is subject. We shall define, rather loosely, a "coverage" by craters in a small size range such that fluxes exceeding the "coverage" value can be called erosive. We then investigate the amount of material worn away, removed, and transported. Estimated rates of meteoritic erosion in space [9.24] are not great, on the order of 10 angstrom units per year. On the lunar surface, the rates are further reduced by the accumulation of a protective layer of debris.[25] That is to say, a uniform "coverage" of an initial rock surface by micro-craters of some size will result in a layer of ejected fragments, probably in a loose and porous form; this layer will impede further attack of the underlying surface. The total thickness of the layer will be augmented by ejection from much larger craters and by some fraction of the infalling primary mass. We now continue with the detailed arguments.

A. FLUX RELATIONS: COVERAGE

The flux of meteoroids has been described above by relations of the form

$$\log N = \log N_o - s \log D \qquad (3.1)$$

where N is the flux of particles per unit area forming craters exceeding diameter D, and N_o and s are constants. This is of course equivalent to the exponential form,

$$N = N_o D^{-s} \qquad (3.2)$$

The flux can also be expressed as a differential quantity n(D), such that:

$$N = \int_D^\infty n(D)\, dD. \qquad (3.3)$$

In the case of the special form, (3.2),

$$+s N_o D^{-s-1} = +n(D) \qquad (3.4)$$

To gain some insight into the nature of erosion, we define the "coverage" of a flux in some time t, as follows. It is the total crater area generated by the influx in that time, per unit area. Coverage is dimensionless. Since no allowance is made for overlap, "coverage" is an upper bound on the fraction of the surface actually occupied by craters; it is greater than one if the surface is "covered" several times. If craters of sizes between D_1 and D_2 are in question, the coverage $C(D_1,D_2)$ is

$$C(D_1,D_2) = t \int_{D_1}^{D_2} n\ (D)\ \frac{\Pi}{4}\ D^2\ dD \qquad (3.5)$$

It is convenient to look at a geometrical range, i.e., $D_2 = K D_1$. If the flux is exponential,

$$C(D,\ KD) = \frac{s}{-s+2}\ \frac{\Pi}{4}\ tN_o D^{-s+2}(K^{-s+2} - 1),\ s \neq 2$$

$$\text{or } \frac{\Pi}{2}\ tN_o \ln K,\ s = 2 \qquad (3.6)$$

Equation (3.6) has some simple and interesting properties. For some reasonable K, say 2, the coverage is approximately the total infall times crater area; that is, from (3.6),

$$tN \cdot \frac{\pi}{4} D^2 = C(D,2D) \left[\frac{1 \quad -s+2}{s \ (2^{-s+2}-1)} \right] \quad s \neq 2,$$

$$= C(D,2D) \left[\frac{1}{2\ln 2} \right] \qquad s = 2. \quad (3.7)$$

The coefficient in brackets is always between 0.6 and 1.0, for S>1. This gives a satisfactory definition for a "severely erosive" infall; if the number density, F, of craters (as on figure 3) exceeds the value, F_e,

$$F_e(D) = Nt = D^{-2}, \qquad (3.8)$$

it is more than 60% probable (Poisson statistics) that any point on the surface lies in a crater of diameter between D and 2D.

The crater densities of figure 3 are compared with this criterion on figure 4. The nominal case is "severely erosive" for D $\leq$10cm, the pessimistic case, for D $\leq$1 meter. The infall then drops, reaching minimum "coverage" near the kilometer level, that is to say, at the precise resolution wherein we judge that the lunar maria are "very smooth". McGillam and Miller(8) quote a continental crater distribution similar to the mare one, but 21 times higher. This population is again becoming "erosive", on the scale of 100's of kilometers; comparison with a lunar photograph shows that our "severely erosive" criterion is conservative.

We conclude that we must consider the cometary influx as the principal eroding agent; the stone influx is also "erosive", but it is dominated by the cometary contribution.

B. Volume Removed

A quantitative estimate of erosion requires calculation of the volume of material "worn away, removed, and transported." As defined above, the volume of a crater of diameter D is

$$\frac{\pi}{12 R} D^3; \qquad (3.9)$$

R = one for hemispherical craters, 2.5 for the preferred 4:1 diameter: depth ratio craters, and attains larger values for very large craters.

The gross volume stirred and transported by an influx in time t for a diameter interval (D_1, D_2) is:

$$V(D_1, D_2) = t \int_{D_1}^{D_2} n\ (D) \frac{\Pi}{12\ R} D^3\ dD. \qquad (3.10)$$

For an exponential influx, this is

$$V(D_1, D_2) = t\ N_o \frac{s}{-s+3} \frac{\Pi}{12\ R} D^{-s+3} \Big|_{D_1}^{D_2} \qquad s \neq 3 \qquad (3.11a)$$

The volume removed is dominated by the upper limit for $s<3$, and by the lower limit for $s>3$. For $s = 3$, (3.11) is

$$V(D_1, D_2) = t\ N_o \frac{\Pi}{4\ R} \ln \frac{D_2}{D_1} \qquad s = 3 \qquad (3.11b)$$

Similar relations may be derived which describe the mass infall on the moon. The study by McCracken and Dubin(25) gives an excellent discussion of the magnitude of the meteoroid flux below the mass limits covered in this paper, and concludes that in 4.5 billion years a total infall of one $gram/cm^2$ has occurred for particles of mass between 10^4 and 10^{-14} grams. (10 and 10^{-17} kg).

Using the cometary flux model (eqn. 2.7) for the whole range, we would overestimate the infall by some orders of magnitude; even with this handicap, our estimate of the net volume of "transported dust" is small. This is so, because most of the dust is transported time and time again.

Consider the differential form of the crater volume opened up by the influx in small ranges of time and diameter.

$$dV = \frac{\Pi}{12\ R}\ n\ (D)\ D^3\ dD\ dt \qquad (3.12)$$

Suppose there is a debris layer of depth, L. Craters of diameter D much smaller than L will only "transport" debris, and not deepen the layer. On the other hand, if $D \gg L$, since dV is the average volume generated per square meter, the mean increase in L is

$$d\ L = PdV \text{ in } m^3/m^2. \qquad (3.13)$$

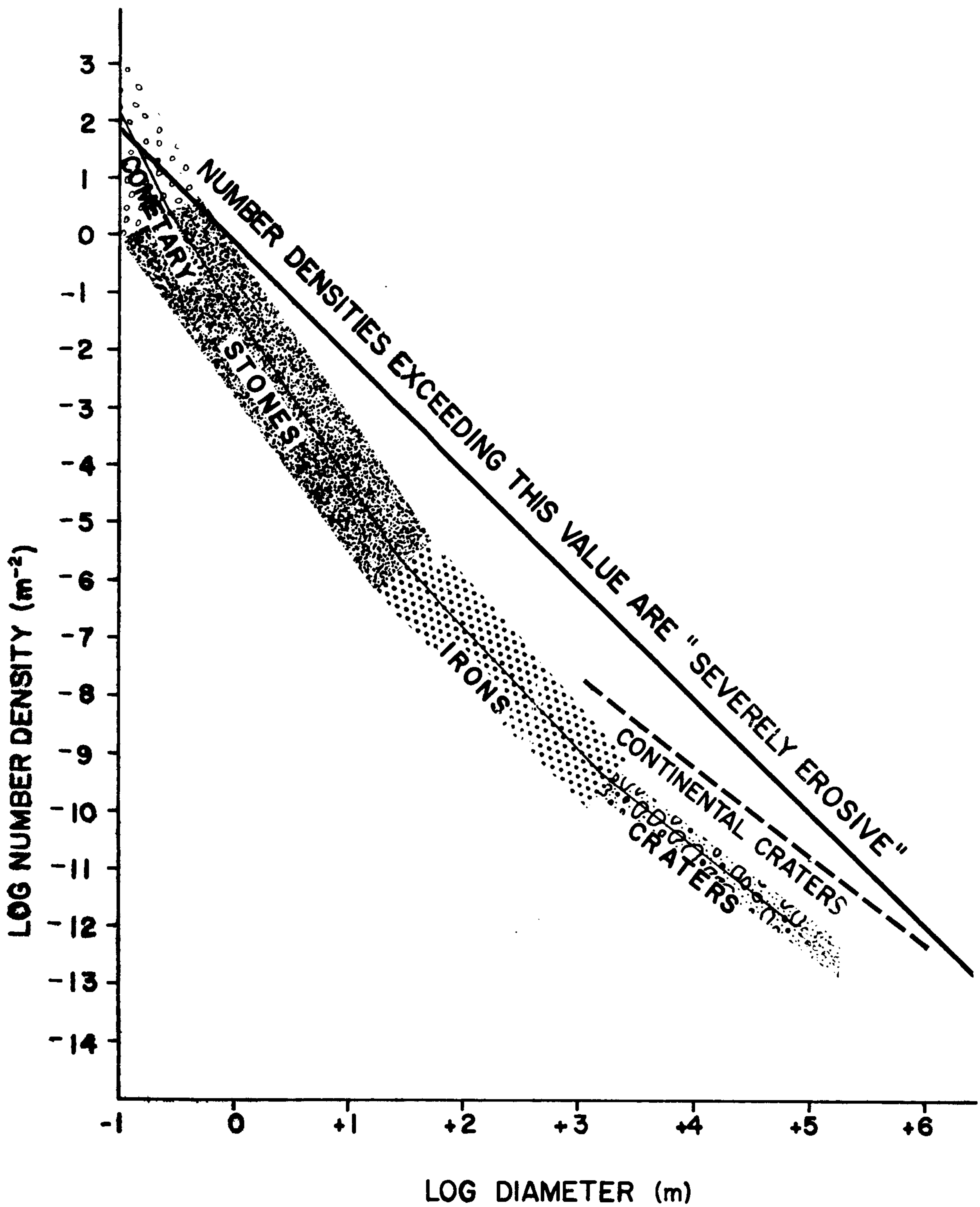


FIG. 4 CRATER DENSITIES AND EROSION CRITERION

The packing number, P, is the ratio of bulk lunar density to layer density. We can take the diameter dependence simply into account by introducing an exponential factor, $e^{-kL/D}$ into (3.12).

$$dL = \frac{P\ \Pi}{12\ R}\ n\ (D)\ D^3\ e^{-kL/D}\ dD\ dt \qquad (3.14)$$

The constant, k, is related to layer strength. The rate of growth of L is then

$$\frac{dL}{dt} = \frac{\Pi P}{12} \int_{\substack{\text{Erosive} \\ \text{Flux}}} \frac{n\ (D)}{R}\ D^3\ e^{-kL/D}\ dD \qquad (3.15)$$

The integral is taken only over the erosive flux because, although volume contributed by the larger craters is very large, the debris is in fact localized near the craters, and does not "cover" the surface.

For an exponential influx, and constant R, (3.15) becomes (we take the entire range of D as "erosive"):

$$\frac{dL}{dt} = \frac{\Pi\ N_o P}{12\ R}\ s \int_0^\infty D^{-s+2}\ e^{-kL/D}\ dD \qquad (3.16)$$

The integral is easy for s = 4; for s > 3 it can be expressed as a gamma function.(26) Let $Z = \frac{kL}{D}$; $D^{-s+2} = (kL)^{-s+2}\ Z^{s-2}$; $dD = -(kL)\ Z^{-2}\ dZ$. The integral becomes

$$(kL)^{-s+3} \int_0^\infty Z^{s-4}\ e^{-Z}\ dZ = (kL)^{-s+3}\ \Gamma(s-3). \qquad (3.17)$$

Here Γ(x) is the gamma function. For positive integers M, Γ(M+1) = M!, Γ(1) = 1. Γ(x) becomes infinite as x approaches zero (in our case, as s decreases towards 3).

Equation (3.16) becomes

$$L^{s-3}\ dL = \frac{\Pi\ N_o\ s\ \Gamma(s-3)\ P\ k^{-s+3}}{12\ R}\ dt \qquad (3.18)$$

which has the solution

$$L^{s-2} = \frac{(s-2)\ s\ \pi\ N_o\ \Gamma(s-3)\ Pk^{-s+3}}{12\ R}\,t \qquad (3.19)$$

The "erosive flux" was identified above as the cometary influx, with $s = 4.02$. The debris layer in this case accumulates nearly as the square root of time. The net accumulation is given by evaluating (3.19) for the age of the maria. The total number density for the cometary infall was given in equation (2.15). In exponential form, this is:

$$F = Nt = N_o t\ D^{-s} = 1.5 \times 10^{-2}\ D^{-4.02} \qquad (3.20)$$

Thus, $N_o t = 0.015$. The estimated error was about 40x. From a table of the gamma function[27], $\Gamma(1.02) = .98884$. We assume 4:1 diameter/depth craters, and take $R = 2.5$. Since the upper surface of the layer is continually bombarded, the majority of the layer should approach close packing. We choose $P = 2$, accordingly. As discussed below, two is also a reasonable value for k. Equation (3.19) becomes

$$L = .11 \text{ meters} \qquad (3.21)$$

with an uncertainty of 6.x, since L depends only on the 1/2.01 power of $N_o t$; errors of the order of 2 or 3x in P and k are swamped by the error in number density.

How good is this estimate? Two questions arise: First, what is the physical meaning of the estimate itself, and second, what other processes contribute to the dust layer?

Firstly, what is the nature of this model layer? Consider the intial differential equation (3.14). If L is zero, the influx removes and transports $\frac{\pi}{12\ R} D^3$ cubic meters of material; for a 4:1 crater, the crater depth is only D/4. The exponential convergence factor (with $k = 2$) then states that for $L = D/2$, a dust layer 2x the depth of the bare-rock crater, 37% of this volume is still removed from base rock. At 5x the depth, 8% is removed. This implies that the crater in the dust layer is 10^2 times the volume of the initial crater.

The model "dust layer", then, is pretty fragile; on the assumption that it is largely close packed (50% porosity), it still comes out 10cm thick at about the 50% confidence level.

Additionally, there is the question of the primary meteoroid infall. As discussed by McCracken & Dubin[25], the meteoroid flux may result in either an accretion or an erosion of the moon. On hard targets, the hypervelocity impact results in a "jet" of very high velocity ejecta, amounting to several particle masses. The jet exceeds the escape velocity of the moon, and a net loss of material results. On the other hand, an extremely porous surface layer will quench the jet and result in a net accumulation of material.

However, as stated above, McCracken and Dubin estimate the total infall (of masses under 10 kg) at 1 gm/cc in 4.5 billion years; in our nominal case of $10^{9 \pm 0.7}$ years, the expected influx is 1/5 gram, or 2mm. The question of either accretion or erosion is then immaterial to the estimated 0.1 meter depth.

The ejecta from larger craters may contribute to the dust blanket, although it does not contribute the "covering" erosion of a particular square meter of surface. This question will be covered in detail in the next section of the paper. Briefly, the contribution from the observable craters is localized near the craters, so that one need worry mainly about the craters between 1 and 10^3 meters in diameter for a contribution to the dust blanket.

For these, the "stone" population is effective (equation 2.17). Since the blanket is not protective against major impacts, we must use Equation (3.11) for the total volume "worn away ... and transported." Substituting, we obtain

$$V(1\ \mathrm{m},\ 10^3\ \mathrm{m}) \quad .134\ \mathrm{m}^3/\mathrm{m}^2 = .134\ \mathrm{m\ depth}, \qquad (3.22)$$

just about doubling our estimate. As will be shown in the next section, much of this volume should appear as isolated blocks of debris, and the net correction of the uniform dust layer thickness is well under a factor of two.

The above analysis assumes a modestly coherent dust quite contrary to Dr. T. Gold's proposal of a very fluid layer(23). Accounting for the erosion products from the degradation of the continental craters, he inferred a flow of fine dust into the maria, where it might accumulate to a depth of 300 feet, "and probably a great deal more." Various authorities, notably H. C. Urey(31), have taken exception to extreme statements of the hypothesis. As pointed out by R. F. Fudali*, the continents possess natural closed basins of deposition which are not filled with marial material. Urey questioned the survival of both color differences and topographic features on the maria, were Gold correct.

*Private Communication

We conclude therefore that a statistical model of the lunar surface may well include a disturbed surface layer of thickness between 2 and 60 centimeters, with confidence about 80%, dependent on the intensity of the primary infall. Even primordial features of size exceeding this should survive.

As for the nature of the layer, the top surface will have been turned over by micrometeoroid bombardment many times in lunar history. The lower levels may be expected to be compacted by this very process. The layer of dust and debris will be deeper in the immediate neighborhood of larger craters. This problem forms the subject of the next section.

IV. DISTRIBUTION OF FRAGMENT SIZES

In this chapter, we discuss the number density and spatial distribution of fragments from the lunar craters. A primary reference for this study is the paper by Gault, Shoemaker, and Moore(28), Spray Ejected from the Lunar Surface by Meteoroid Impact. We shall refer to this paper below as GSM. Most of the empirical background for the section, plus some very similar analysis, will be found there. Since the emphasis here will be on surface topography, the approach is somewhat different; and the analytic study is less closely dependent on the empirical impact results than that in GSM.

Each major impact results in a crater of some volume. As described by Baldwin(5) a substantial portion of this is accounted for by deformation of the substrate, the surrounding terrain being squeezed and folded up to form the rim. The remainder is projected into space with varying velocities, as described by GSM. It is hard to estimate the portion ejected; we shall generally use the entire crater volume. Of the ejecta a few projectile masses (depending on the impact velocity) exceed 2.4 km/sec., and leave the Moon. The remainder (some hundreds of projectile masses) is redeposited , mostly within a few km of the crater. The question to be asked here is: compared with the small (1 meter) crater population, what has been the incidence of fragments of the same general size? If this can be obtained, some of the fragments will survive as such; others will have generated secondary craters. Since the velocity of secondary impact is low, the contribution of terrain roughness should be about the same in either case.

To attack the problem at all, we must have the primary infall rates estimated in chapter II, and a comminution (grinding) relation which relates the fragment sizes to the size of the parent crater. The empirical distributions presented by GSM are exponentials, given in the form,

$$\frac{M_e}{M} = \left(\frac{e}{b}\right)^{\alpha} \tag{4.1}$$

M is the total ejected mass from the crater; M_e is the cumulative mass of fragments smaller than e in diameter; and b is the diameter of the largest fragment, typically about a tenth of the crater diameter, but less than this for very large craters. Empirically, α ranges from 0.3 to 0.7. GSM use a value of 0.4. The crucial element of the comminution scheme (4.1) is the approximate scaling with crater diameter; there are, for instance, about the same number of fragments of diameters D/100 for all crater diameters, D. Some very interesting results require no further assumption than this.

It is a rather strong assumption; but the results are still useful if the fragment distribution changes only slowly over the range of crater diameters concerned.

The assumption can be expressed as follows. From a primary crater of diameter D, a number of secondary fragments are thrown out. The cumulative number of secondaries exceeding diameter e is $N_s(e)$; the number between diameters e and e + de is assumed a function only of e/D. Thus,

$$dN_s(e) = A \circ g(e/D) \cdot d(e/D) \tag{4.2}$$

The coefficient A is a constant. The function g can be arbitrary, within the condition that the total volume of secondaries corresponds with that of the ejected crater material. The volume contained in the dN_s secondaries is

$$dV(e,D) = A\frac{\Pi}{6}\, e^3 g(e/D) d(e/D). \tag{4.3}$$

For simplicity, we have assumed spherical fragments of diameter e. The total volume in fragments smaller than e is:

$$V(e,D) = A\frac{\Pi}{6}D^3 \int_0^{e/D} y^3\, g(y)\, dy. \tag{4.4}$$

The variable $y = (e/D)$ has been substituted. We now put on the requirement that the secondary volume equal the crater ejected volume, V. Let this be

$$V = \frac{\Pi D^3}{12R_e} = A\frac{\Pi}{6}D^3 \int_0^1 y^3 g(y)dy \qquad (4.5)$$

Above, we defined $\Pi D^3/12R$ as the volume of a primary crater; $\Pi D^3/12R_e$ is the volume ejected. As stated above we shall generally set $R_e = R$, assuming that the entire crater volume is ejected. From (4.5), we can set a value for A. It is convenient if $A = (1/2R_e)$; then,

$$\int_0^1 y^3 g(y)dy = 1. \qquad (4.6)$$

Considering (4.4) again, V(e,D) must go to zero with e; it is useful to set limits on the behavior of g(y) which are consistent with (4.1). We write

$$\lim_{e\to o} \frac{V(e,D)}{V} \leq \left(\frac{e}{D}\right)^{\alpha} \qquad \alpha = 0.4 \qquad (4.7)$$

By differentiation of (4.4), we see that a sufficient condition for this is:

$$\lim_{y\to o} \quad g(y) < \alpha y^{\alpha-4} \qquad (4.8)$$

We can now compute the secondary particle flux in terms of the primary influx described in the last chapters. Since our influx is already expressed in crater diameters, the result is quite different in form from that obtained in GSM. The number of secondary particles exceeding diameter e resulting from $dN_p(D)$ primary craters is:

$$dN_s(e) = \frac{1}{2R}dN_p(D) \int_{e/D}^1 g(y)dy \qquad (4.9)$$

As in the preceding chapter Equation 3.3, we take the differential distribution of craters as n(D).

The total number of secondary particles exceeding diameter e is:

$$N_s(e) = \frac{1}{2R} \int_e^\infty n(D)dD \int_{e/D}^1 g(y)dy \qquad (4.10)$$

Something should be said about the limits of the D integral. When using some fixed empirical form for g, it is necessary to start the D integration at the D value greater than e which first produces e-size fragments. With g arbitrary, we can imagine it identically zero for the appropriate range of y, from about 0.1 to 1.

We now invert the order of integration in (4.10)

$$N_s(e) = \frac{1}{2R} \int_0^1 g(y)dy \int_{e/y}^\infty n(D)dD. \qquad (4.11)$$

This is, by the definition of $N_p(D)$,

$$N_s(e) = \frac{1}{2R} \int_0^1 g(y)N_p(e/y)dy \qquad (4.12)$$

Now in chapter two the crater distribution was represented by a sum of several exponential fluxes (Equation 3.2), each dominating in a particular diameter interval. This could be written as a summation,

$$N_p(D) - \sum_i N_i D^{-s_i} \qquad (4.13)$$

Each term will contribute secondaries; the result is,

$$N_s(e) = \frac{1}{2R} \sum_i N_i e^{-s_i} \int_0^1 g(y)y^{s_i}\ dy \qquad (4.14)$$

That is, each component of the primary flux contributes a distribution of ejected fragments which save for a constant has the same exponential size distribution as the primaries.

For the stone meteorite infall (2.17) $s_i = 3$. But the integral is normal for this value (equation 4.5)

$$N_s(e) = \frac{1}{2R} N_p(e) \qquad s = 3 \qquad (4.15)$$

This means that volume for volume, there are equal numbers of primary craters and secondary fragments. That is, a 1 meter spherical fragment has 2R times the ejected volume of the 1 meter crater. The number density of 1 m fragments is therefore reduced 2R times.

For steeper fluxes (for instance, the cometary flux, (2.15)) the integral is less than one. We can show this in a special case which doesn't greatly affect the arbitrariness of the function g(y). Suppose that, in accord with the GSM results, there is no fragment larger than qD in size, where q is about a tenth. Then g(y) must be zero for $y>q$. Since y^s is an increasing function for y and s positive, the following inequality holds,

$$\int_0^1 g(y)y^s dy = \int_0^q g(y)y^3y^{s-3}dy$$

$$< q^{s-3}\int_0^1 g(y)y^3 dy = q^{s-3} \qquad (4.16)$$

so, for $s>3$,

$$N_s(e) < \frac{q^{s-3}}{2R} N_p(e), s>3 \qquad (4.17)$$

In the same way, it can be shown that for $s<3$, the fragment population increases more rapidly than q^{s-3}.

This leads us to the question of what happens to the distribution when s reaches the values of 2.1 (for the irons) or 1.50 (for the lunar craters).

The general arguments we have used heretofore are less useful in this range. It becomes necessary to use detailed models; this can be done much more efficiently by the experts in the field. However, it should be noted that the throwout from craters is localized. Gault, Shoemaker, and Moore estimate that 50% of the ejected mass travels less than a kilometer, and 90% less than thirty kilometers. Thus, estimates of uniform debris density should not be based on craters less dense than perhaps one every 10^6 m^2. This density occurs at about the change to smaller s values. We can then properly speak of the uniform debris density as being given by equation (4.15) and (4.17).

The dominant infall in the 1 - 100 meter diameter interval is that of the stone meteoroids, whose number density is given by equation (2.17). We can then derive the number density of ejecta fragments in the 1 meter size range; we take R = 2.5 (4:1 craters), and obtain for the cumulative number per square meter exceeding diameter e,

$$\log F(e) = -1.91 - 3\log e \pm 1.3. \qquad (4.18)$$

For the larger craters the nature of the comminution relation must be specified; this can be studied both from the laboratory impact experiments (as in GSM) and by studies of lunar topography, as in E. M. Shoemaker's article on the Interpretation of Lunar Craters.[29]

In conclusion, under our assumption of a uniform scaling law, we expect a reasonably uniform density of secondary fragments (or satellitic craters) such that for each primary crater there is a secondary element of equivalent volume. Primary craters of number density exceeding one per square kilometer contribute to this distribution; larger and less frequent craters must be considered as surrounded by localized debris concentrations, which may, however, be substantial at distances of thirty kilometers from the crater lip.

V. MODEL OF LUNAR SURFACE ROUGHNESS DERIVED FROM METEORIC INFALL: IMPLICATIONS AND CONCLUSIONS

In the above pages we have endeavoured to produce a sequence of lunar surface models, on the hypothesis that lunar topography has resulted solely from meteoric impacts. It is pertinent to mention that the earth's topography is extraordinarily varied and has resulted from a large number of processes among which meteoric infall must be considered rather minor. It is only reasonable that lunar topography as well should owe its origin to a number of processes, constructive and destructive. Meteoric infall is surely more important than on earth; it remains only part of the story. It is, however, a part on which reasonably quantitative roughness estimates can be made, and this is its value. We now summarize the results obtained above and briefly examine some of the simpler implications for lunar missions.

In chapter II, the meteoroid influx was studied. It appeared that in the range of 0.1 to 10 meter crater sizes, the dominant infall was that of the brilliant visual meteors, or the stone meteorites. We expressed the inferred cumulative number density of craters exceeding diameter D in meters, as F, per square meter on the lunar maria.

$$\log F = -1.21 - 3 \log D \pm 1.3 \qquad (2.17)$$

The uncertainty of twenty times reflected lack of knowledge about the meteoroid flux, its constancy over the debated age of the maria, and about the crater size associated with a given impact. The craters were assumed to have diameter to depth ratios of 4:1. In chapter IV, we showed, assuming a uniform comminution law relating fragment size to crater diameter, that to each primary crater there would correspond one fragment of equivalent volume. Related to the primary crater density above, the density of (spherical) fragments of diameter e or greater would be

$$\log F(e) = -1.91 - 3 \log e \pm 1.3 \qquad (4.18)$$

This estimate includes the entire contribution from the infall of stone meteoroids. The contribution from the flux of iron meteoroids dominates the number densities of primary craters of diameters exceeding about 100 meters. The substantial debris concentration from these is not contained in (4.18), in part because these craters are infrequent enough that the rubble may be associated with a given crater, rather than being uniformly distributed.

In chapter III, the degree to which features were degraded by meteoritic erosion was considered. It was concluded that the attack of bedrock by smaller particles would soon be terminated by the accumulation of a protective debris layer. The total accumulation of this layer due to the entire "cometary influx" was estimated as

$$L = .1 \text{ meters} \qquad (3.21)$$

with an uncertainty of six times. The layer would not protect against more massive impacts, but we estimated that throwout from these would not double layer thickness, and was unimportant within estimated error. The upper regions of the layer are being continually "turned over" by bombardment by the smallest meteoroids. The shock transmitted down can be assumed to compact the lower levels to a structurally satisfactory degree. The total depth cannot be considered an "incompetent dust".

Let us now summarize the models of lunar surface roughness: It will be remembered that the "models" can be best considered as upper bounds to the numbers tabulated, with confidences of perhaps 90% (pessimistic), 50% (nominal) and 10% (optimistic).

In table II we tabulate, for these models, the dust layer depth, the cumulative density of one meter craters (depth 0.25m) and the cumulative density of 1 meter diameter spherical fragments. The latter offer much more danger to spacecraft.

We now very briefly investigate the implications of these models for lunar missions. Generally, these models are quite rough.

The pessimistic case is quite rugged. The surface is "covered" by 1 meter craters, and should--provided the dust layer was absent--resemble the continental regions of the moon as seen on photographs. One meter fragments are on the mean spaced every 2 meters. The dust layer is of depth 70 centimeters; no doubt this will conceal many features. We cannot quantitatively estimate sinkage, but, clearly, dependent on underlying topography, sinkage approaching 20 or 30 centimeters is plausible. Withal, the model is not hopelessly inhospitable. Such a boulder - strewn landscape would not be too difficult to walk on. The consistent ruggedness might make it plausible for landing; the surface would have to be essentially uniformly covered with crushed rock. There would be no question of finding a "smooth area" other than one where the dust might conceal unknown topography.

The optimistic case is quite another matter. Two centimeters of "dust" cover the surface, and sinkage exceeding one is unlikely. One 1 meter crater is expected every 300 m^2; using the Poisson formula, one has about a 35% chance of finding a 20 meter diameter circle free of craters larger than 1 meter (and an 80% chance of finding one free of 1 meter ejecta).

That this is an "optimistic" case is rather a shock if one has seriously considered requiring a landing site roughly a square kilometer in area to be free of such obstacles. Roughly 3000 craters and 600 fragments exceeding 1 meter in size are "expected" in this area. Clearly, however, the model offers little obstacle to a skillful astronaut landing a vehicle which is docile and has good visibility.

In the nominal case, the 12 centimeter dust depth (sinkage probably under 6) is allowable. The chances of finding a 20 meter diameter circle free of craters are vanishing; the chance is but 2% to find it free of rocks.

It then appears that the surfaces presented in these models are not terribly encouraging. The burden of landing safety

TABLE II

MODELS

METEORIC INFALL AND LUNAR SURFACE ROUGHNESS
CRATER AND FRAGMENT COUNTS SCALE AS THE CUBE
OF FEATURE DIMENSION.

	Density of features exceeding 1 meter (m^{-2}).		"Dust Layer"(m)
	Craters (4.1)	Fragments	Depth
Pessimistic	1.2	.26	.70
Nominal	.06	.013	.12
Optimistic	.003	.0006	.02

may rest strongly on the evidence of a reconnaissance mission, either unmanned or manned. There are good reasons why one region of the moon might be preferable to another.

There is still question, for instance as to the relative ages of the various maria. Though Shoemaker et al[6] estimated that the maria were closely the same age, Fielder[30] has proposed that some may be very much younger than others. Since both investigations compared crater counts, this should be looked into. Darker and lighter areas in the maria have been associated with varying age. Terrestrial nuclear explosion craters are relatively smooth. It is conceivable that the interior of a recent post-marial crater might be a superior landing site.

With regard to site location and verification, it is most significant that we expect the crater population to follow a cube law in the range of interest. Thus, we would feel quite good confidence in extrapolating crater counts obtained by an orbiting spacecraft; supposing that it could define a distribution between diameters of 50m and 5m, we would expect (depending of course on the nature of the data) something like a factor of 3 accuracy at 1 meter. If this were to be spot-checked by a very few landed Surveyors, extrapolation to 0.5 meter could be justified with useful confidence over broad areas.

In closing, it is a matter of considerable regret that we are unable to support the so-called, "Daytona Beach" model of the lunar surface. These smooth, strong, desert-like expanses beloved by science-fiction illustrators seem excluded from consideration. Nature, of course, is under no compulsion to be hospitable to man; quite to the contrary. we have spent some billion years adapting ourselves to her.

The extension of man's living space to the moon only continues this adaptive process. If the models presented here can encourage a reasoned approach to this matter, they will have served a useful purpose.

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REFERENCES

1. Fielder, G., Structure of the Moon's Surface, Pergamon Press, New York, 1961.

2. Lunar Aeronautical Charts, U. S. Air Force; available from the Superintendant of Documents, Washington, D. C.

3. H. Moore, private communication.

4. Shoemaker, E. M., American Scientist, vol. 50, 1, pp. 99-130.

5. Baldwin, R. B., Measure of the Moon, University of Chicago Press, Chicago, Illinois, 1963.

6. Shoemaker, E. M., Hackman, R. J., and Eggleston, R. E., Interplanetary Correlation of Geological Time, U. S. Department of Interior open file report, prepared for 7th annual meeting of the American Astronautical Society.

7. Opik, E. J., Monthly Notice of the Royal Astronomical Society, 120, pp. 404-411, 1960.

8. McGillem, C. P., and Miller, B. P., J. Geophys, Res, vol. 67, 12, November 1962, pp. 4787-4794.

9. Orrok, G. T., The Meteoroid Environment of Project Apollo, Bellcomm, Inc., January 31, 1963.

10. Hawkins, G. S., Nature, vol. 197, February 23, 1963, p. 781.

11. Proceedings of the Fifth Symposium on Hyper Velocity Impact, Denver, Col., October 1961, (Tri Service Committee, contract n° Nonr (G)-0020-62(X).) April 1962. (The Proceedings of the Sixth Symposium should be available soon.)

12. Bjork, R. L., Rand Corp. Report P1662, December 16, 1958.

13. Charters, A. C. and Locke, G. S. Jr., NACA Report A58B26, 1958.

14. Hawkins, G. S., and Upton, E. K. L., Astrophys, J., vol. 128, 3, 1958, pp. 727-735.

15. Opik, E. J., Physics of Meteor Flight in the Atmosphere, Inter-science, New York, 1958.

16. Levin, B. J., The Physical Theory of Meteors, (Chapters I-III), translated, American Meteorological Society, Astia, AD110091.

17. Whipple, F. L., On Meteoroids and Penetration, JGR, 68, 17, 4929-4939, September 1, 1963.

18. Hawkins, G. S., Astronom, J., vol. 64, 1275; 1959, pp. 450-454.

19. Hawkins, G. S., Astronom J., vol. 65, 318; 1960.

20. Brown, H. S., J. Geophys Res. vol. 65, 6; 1960, pp. 1679-1683, with addendum, J. Geophys Res. vol. 66, 4; 1961, pp. 1316-1317.

21. Millard, H. T. Jr., J. Geophys Res. vol. 68, 14, 4297-4303, 1963.

22. Jacchia, L. and Whipple, F. L., Precision Orbits of 413 Photographic Meteors, Smithsonian Contribution to Astrophysics, vol. 4, 4.

23. Gold, T., M.N., 115, 585-604, 1955.

24. Whipple, F. L., Smithsonian Contributions to Astrophysics, vol. 7, 1963, pp. 239-248.

25. McCracken, C. W., and Dubin, M., Dust Bombardment on the Lunar Surface, presented at the Lunar Surface Materials Conference, Boston, Mass., May, 1963.

26. See any advanced calculus test, for instance: Widder, D. V. Advanced Calculus, Prentice Hall, New York, 1947, p. 303.

27. Handbook of Chemistry and Physics, 43rd Edition, Chemical Rubber Publishing Co., Cleveland, Ohio, p. 285.

28. Gault, D. E., Shoemaker, E. M. and Moore, H. J. Spray Ejected From the Lunar Surface by Meteoroid Impact NASA TND-1767, April, 1963.

29. Shoemaker, E. M. Interpretation of Lunar Craters in Physics and Astronomy of the Moon, Z. Kopal, ed., Academic Press, New York, 1961, pp. 283-359.

30. Fielder, G., Nature, vol. 198, June, 1963, pp. 1256-1260.

31. Urey, H. C., Observatory, 76, 212-213, December, 1956, Sky and Telescope, 15, 3, 108-111, January, 1956 and 4, 161-163, February, 1956.

LIST OF SYMBOLS

Each symbol is followed by the number of the equation in which it first appears:

A (4.2) a constant

b (4.1) diameter of the largest fragment from a given impact

$C(D_1,D_2)$ (3.5) "coverage;" fraction of area covered by craters of diameters between D_1 and D_2

c (2.2) sound velocity

D (2.1) crater diameter

D_1,D_2 specific values of D

d (2.2) particle diameter for a primary meteoroid

e_x,e_y,e_z,e_{xyz} (1.1) errors in quantities x, etc.

e (4.1) diameter of an ejected fragment

F(D) (2.1) cumulative number density of craters per unit area

$F_e(D)$ (3.8) a cumulative number density of craters which is "severely erosive." see definition at (3.8)

g(e/D) (4.2) a function describing the size distribution of fragments from an impact

i (subscript, (4.13): an integer)

K (3.6) a constant

k (3.13) a constant related to layer strength

L (3.13) layer depth

ln: natural logarithm

log: logarithm to base 10

M (4.1) total ejected mass

m (2.6) primary particle mass

M_e (4.1) cumulative mass of fragments smaller than e, from one crater

N(D) (2.6) cumulative flux, per square meter second of particles forming craters exceeding diameter D

N_p (4.9) primary flux

N_o (3.1) a constant in a flux equation

N_i (4.13) one of a set of numerical values for N_o

N_s (e) (4.2) cumulative flux of secondaries exceeding diameter e

n(D) (3.3) differential flux of primaries

P (3.13) packing number. The ratio of base material density to layer density

p (2.2) penetration in a semi-infinite solid

q (4.15) ratio of largest particle diameter to crater diameter, b/D

R (2.5) volume ratio: hemispherical crater to real crater

R_e (4.6) volume ratio: hemispherical crater to ejected volume

S crushing strength of a target (following (2.2))

s (3.2) population coefficient, exponent of diameter in cumulative flux law against diameter

s_i (4.13) one of a set of numerical values for s

t (3.5) time

$V(D_1,D_2)$ (3.10) volume eroded /m^2 by total infall in the diameter range (D_1,D_2)

V(e,D) (4.4) total volume of ejecta of diameters less than e from a crater of diameter D

V_e (4.6) total volume of ejecta from crater of diameter D

v (2.2) primary particle velocity

x,y,z dummy variables

α (4.1) an exponent in an empirical scaling law

$\Gamma(x)$ (3.17) the gamma function

ρ_p (2.2) density of primary particle

ρ_t (2.2) density of target